# On Property (GR)

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***Abstract***-*In this paper we introduce and study property (GR) for bounded linear operators defined on a Banach space related to Weyl type Theorems.*
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## 1. Introduction

Throughout this paper, X denotes an infinite-dimensional complex Banach space and B(X), the algebra of all bounded linear operators on X. for $T \in B(X)$, let $T^*$, N(T), R(T), $\sigma(T)$ and $\sigma a(T)$ denote the adjoint, null space, the range, the spectrum and approximate point spectrum of T, respectively.
If $\alpha(T) := \dim \ker T < \infty$ and R(T) is closed then $T \in B(X)$ is said to be upper semi-Fredholm operator while $T \in B(X)$ is lower semi-Fredholm if $\beta(T) := \operatorname{codim} R(T) < \infty$. The index of semi-Fredholm operator is defined as $\operatorname{ind} T := \alpha(T) - \beta(T)$. An operator $T \in B(X)$ is said to be semi-Fredholm operator if T is either as upper or a lower semi-Fredholm operator. Let $\phi(X)$, $\phi_+(X)$ and $\phi\_(X)$ denote the classes of Fredholm operators, upper semi-Fredholm operators and lower semi-Fredholm operators, respectively.

An operator T is called Weyl if it is a Fredholm operator of index zero. For
$T \in B(X)$, let
$W_+(X) = \{T \in \phi+(X), \operatorname{ind} T \leq 0\}$,
$W\_(X) = \{T \in \phi\_(X), \operatorname{ind} T \geq 0\}$
The class of Weyl operators $W(X) := W_+(X) \cap W\_(X) = \{T \in \phi(X), \operatorname{ind} T = 0\}$.

These classes of operators generate the following spectra:
The Weyl spectrum
$\sigma_w(T) := \{\lambda \in C : T - \lambda 1 \notin W(X)\}$,
the upper semi-Weyl spectrum (or Weyl essential approximate spectrum)
$\sigma_{uw}(T) := \{\lambda \in C : T - \lambda 1 \notin W_+(X)\}$,
the lower semi-Weyl spectrum (or Weyl essential surjective spectrum)
$\sigma_{\ell w}(T) := \{\lambda \in C : T - \lambda 1 \notin W\_(X)\}$,
obviously $\sigma_w(T) = \sigma_{uw}(T) \cup \sigma_{\ell w}(T)$ and from classical Fredholm theory we have
$\sigma_{uw}(T) := \sigma_{\ell w}(T^*)$, $\sigma_{\ell w}(T) = \ell_{uw}(T^*)$
For an operator $T \in B(X)$, the ascent is defined as $p := p(T) = \inf\{n \in N : \ker T^n = \ker T^{n+1}\}$ while the descent is defined as $q := q(T) = \inf\{n \in N : T^n(X) = T^{n+1}(X)\}$. If p(T) and q(T) are both finite, then $p(T) = q(T)$. If $0 < p(T - \lambda I) = q(T - \lambda I) < \infty$, then $\lambda$ is a pole of the resolvent of T. The class of all upper semi-Browder operator is defined as $B_+(X) := \{T \in \phi_+(X) : p(T) < \infty\}$, the class of lower semi-Browder operator in defined as $B\_(X) = \{T \in \phi\_(X) : q(T) < \infty\}$. The class of all Browder operators is
$B(X) := B_+(X) \cap B\_(X) = \{T \in \phi(X) : p(T) = q(T) < \infty\}$.
Evidently

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$B(X) \subseteq W(X)$, $B_+(X) \subseteq W_+(X)$, $B\_(X) \subseteq W\_(X)$.
The definitions lead us to the following spectra:
The Browder spectrum
$\sigma_b(T) = \{\lambda \in C : T - \lambda 1 \notin B(X)\}$;
the upper semi-Browder spectrum is de_ned by
$\sigma_{ub}(T) = \{\lambda \in C : T - \lambda 1 \notin B+(X)\}$

For $T \in B(X)$ and a non-negative integer n, define $T_n$ to be the restriction of T to $R(T^n)$ viewed a map from $R(T^n)$ onto $R(T^n)$ (in particular $T_0 = T$). If for some integer n the range space $R(T^n)$ is closed and $T_n$ is an upper (resp, a lower) semi-Fredholm operator, then T is called an upper (resp, a lower) semi. B-Fredholm operator. In this case the index of T is defined as the index of the semi B-Fredholm operator. Moreover, if Tn is a Fredholm operator, then T is called B-Fredholm operator. An operator $T \in B(X)$ is said to be a B-Weyl operator if it is a B-Fredholm operator of index zero. Let $SBF_{\mp}(X)$ be the class of all upper semi-B-Fredholm operator
$SBF_{\mp}(X) = \{T \in SBF_+(X) : \text{ind}(T) \leq 0\}$
This generates the following spectra:
The upper B-Weyl spectrum of T
$\sigma_{SBF_{\mp}}(T) = \{\lambda \in C : T - \lambda 1 \notin SBF_{\mp}(X)\}$

The B-Weyl spectrum
$\sigma_{BW}(T) = \{\lambda \in C : T - \lambda I \text{ is not B-Weyl operator}\}$.

An operator $T \in B(X)$ is called Drazin invertible if it has a finite ascent and descent. The Drazin spectrum $\sigma_D(T)$ of an operator T is defined by
$\sigma_D(T) := \{\lambda \in C : T - \lambda I \text{ is not Drazin invertible}\}$.
Define $LD(X) := \{T \in B(X) : p(T) < \infty$ and $R(T^{p(T)+1})$ is closed$\}$ and
$\sigma LD(T) = \{\lambda \in C : T - \lambda 1 \in LD(X)\}$ we say $\lambda \in \sigma_a(T)$ is left pole of T if $T - \lambda I \in LD(X)$ and $\lambda \in \sigma_a(T)$ is a left pole of T of finite rank if $\lambda$ is a left pole of T and $0 < \alpha(T - \lambda I) < \infty$. Let $\pi^a(T)$ denote the set of all left poles of T and $\pi_0^a(T)$ denote the set of all left poles of finite rank.
Let $\pi(T)$ be the set of all poles of the resolvent of T and $\pi^0(T)$ be the set of all poles of the resolvent of T of finite rank. It is obvious that $\lambda$ is a pole of T if and only if $\lambda$ is both a left and a right pole of T. In fact, if $\lambda$ is a pole of T then $\lambda I - T$ is Drazin invertible, so $\lambda I - T$ is both left and right Drazin invertible. Moreover $\lambda$ is both left and right pole of T, since $0 < p(\lambda I - T) = q(\lambda I - T) < \infty$ entails that $\lambda \in \sigma_a(T)$ as well as $\lambda \in \sigma_s(T)$.
A bounded linear operator $T \in B(X)$ is said to be left polaroid if every isolated point of $\sigma_a(T)$ is a left-pole of the resolvent of T. $T \in B(X)$ is said to be right polaroid if every isolated point of $\sigma_s(T)$ is a right pole of the resolvent of T. $T \in B(X)$ is said to be polaroid if every isolated point of $\sigma(T)$ is a pole of the resolvent of T. A bounded linear operator $T \in B(X)$ is said to be a-polaroid if every $\lambda \in \text{iso}\, \sigma_a(T)$ is a pole of the resolvent of T [2]. Thus,
T a polaroid $\Rightarrow$ T left polaroid
and
T a polaroid $\Rightarrow$ T polaroid.
If $T \in B(X)$. Define
$E(T) := \{\lambda \in \text{iso}\, \sigma(T) : 0 < \alpha(T - \lambda I)\}$,
$E_0(T) := \{\lambda \in \text{iso}\, \sigma(T) : 0 < \alpha(T - \lambda I) < \infty\}$.
Then, for every $T \in B(X)$
$\pi_0(T) \subset \pi(T) \subset E(T) \subset E^a(T)$ and $\pi_0(T) \subset \pi_0^a(T) \subset E_0^a(T)$.
We say that Weyl's theorem holds for T if $\sigma(T) \sim \sigma_w(T) = E_0(T)$; generalized Weyl's theorem holds for T if $\sigma(T) \sim \sigma_{BW}(T) = E(T)$; a- Weyl's theorem holds for T if $\sigma_a(T) \sim \sigma_{uw}(T) = E_a 0 (T)$; generalized a- Weyl's theorem holds for T, if
$\sigma_a(T) \sim \sigma SBF_{\mp}(T) = E_a(T)$; Browder's theorem holds for T if $\sigma(T) \sim \sigma_w(T) = \pi 0(T)$; a-Browder's theorem holds for T if $\sigma a(T) \sim \sigma uw(T) = \pi_a 0 (T)$; generalized a-Browder's theorem if
$\sigma_a(T) \sim \sigma SBF_{\mp}(T) = \pi a(T)$. Property (w) holds if $\sigma(T) \sim \sigma_{uw}(T) = E_0(T)$ Property (gw) holds if $\sigma a(T) \sim \sigma SBF_{\mp}(T) = E(T)$.

The following property has important role in local spectral theory [1]

Definition 1.1. Let X be a complex Banach space and $T \in B(X)$. The operator T is said to have the single valued extension property $\lambda_0 \in C$ (abbreviated SVEP at $\lambda_0$), if for every open disc D of $\lambda_0$, the only analytic function $f : D \to X$ which satisfies the equation $(\lambda I - T)f(\lambda) = 0$ for all $\lambda \in D$ is the function $f \equiv 0$.

An operator $T \in B(X)$ is said to have SVEP if T has SVEP at every point $\lambda \in C$. Evidently, $T \in B(X)$ has SVEP at every isolated point of the spectrum. we have

$p(\lambda I - T) < \infty \Rightarrow T$ has SVEP at $\lambda$, (1)

and

$q(\lambda I - T) < \infty \Rightarrow T^*$ has SVEP at $\lambda$. (2)

Furthermore,

$\sigma_a(T)$ does not cluster at $\lambda \Rightarrow T$ has SVEP at $\lambda$, (3)

$\sigma_s(T)$ does not cluster at $\lambda \Rightarrow T^*$ has SVEP at $\lambda$ (4)

All implication (1)-(4) are equivalent whenever $T - \lambda I$ is a semi B-Fredholmoperator.

In [4] the property (R) is defined and studied for bounded linear operator.

An operator $T \in B(X)$ satis_es property (R) if $\pi_0^a(T) = E_0(T)$. It means T satisfies property (R) if the isolated points of the spectrum $\sigma(T)$ of T which are eigenvalues of finite multiplicity are exactly those points $\lambda$ of the approximate point spectrum for which $\lambda I - T$ is upper semi-Browder. It is shown left property (R) is strictly related to property (w) introduced by Rakocevic in [13] and more extensively studied in recent papers ([3], [1]). In this paper our aim is to introduce property (GR) which in related to generalized a-Browder's Theorem. We shall study this property using a localized version of the single-valued extension property and in the framework of a-polaroid operators.

## 2. Property (GR)

We say an operator $T \in B(X)$ satisfies property (GR) if $\pi_a(T) = E(T)$ holds.

The next result shows that property (GR) can be studied as half of the property (gw) in the following way:

**Theorem 2.1.** T satisfies property (gw) if and only if generalised a-Browder's theorem holds for T and T has property (GR).

The following example is given in support of Theorem 2.1:

**Example 2.2.** Let $R \in B(\ell^2(N))$ be the unilateral right shift and U defiined by

$U(x_1, x_2, \ldots) = (0, x_2, x_3, \ldots), (xn) \in \ell^2(N)$.

If $T = R \oplus U$ then $\sigma(T) = D(0, 1)$ the closed unit disc in C, iso $\sigma(T) = \phi, E(T) = \phi$ and $\sigma_a(T) = C(0, 1) \cup \{0\}$ where C(0, 1) is unit circle in C.

$\sigma SBF_{\mp}(T) = C(0, 1)$ Thus, $\sigma_a(T) \sim \sigma SBF_{\mp}(T) = \{0\} \neq E(T)$. Property (gw) and property (GR) are not satisfied, but T satisfies generalised a-Browder's Theorem.

**Theorem 2.3.** If T satisfies property (GR) and $N(T - \lambda I) < \infty$ for all $\lambda \in$ iso $\sigma(T)$, Then T satisfies property (R).

The following example shows that property (R) is weaker than property (GR).

**Example 2.4.** Let $T \in B(\ell^2(N))$ be the weighted right shift operator defined by

$T(x_1, x_2, \ldots) := (0, x_{1/2}, x_{2/3}, \ldots)$, for all $x = (x_1, x_2, \ldots) \in \ell^2(N)$ Then $\sigma_a(T) = \{0\}$, $\pi_0^a(T) = \phi$, $E_0(T) = \{0\}$. Then T does not satisfy property (R). But $\pi_a(T) = \{0\} = E(T)$. So T satisfies property (GR).

**Theorem 2.5.** If $T \in B(X)$, then

(i) T satisfies property (GR) if and only if E(T) coincides with the set of left poles of T.

(ii) T* satisfies property (GR) iff E(T_) coincides with the set of right poles of T.

(iii) T satisfies property (GR) then $E(T) = \pi(T)$

Proof. (i) and (ii) follow from the definitions

(iii) we know $\pi(T) \subset E(T)$ for all $T \in B(X)$. To show the opposite inclusion, suppose that T satisfies property (GR) and let $\lambda \in E(T) = \pi_a(T)$ then $p(T - \lambda I) < \infty$. Since $\lambda \in$ iso $\sigma(T)$, then T*has SVEP at $\lambda$ [1, Theorem 2.47] and $\lambda I - T$ is upper semi-Browder and $q(\lambda I - T) < \infty$. Thus, $\lambda$ is a pole of the resolvent of T. Thus, $\lambda \in \pi(T)$. Hence E(T)

$= \pi(T)$. _The quasi nilpotent part H0(T −λI) of (T −λI) of (T −λI) is defined by

$H_0(T-\lambda I) = \{x \in X : \lim_{n\to\infty} \|(T-\lambda I)^n x\|^{1/n} = 0\}$

**Theorem 2.6.** An operator $T \in B(X)$ satisfies property (GR) if and only if the following two conditions hold:

(i) $\pi_a(T) \subseteq \text{iso}\, \sigma(T)$

(ii) $\dim H_0(T-\lambda I) < \infty$ for all $\lambda \in E(T)$.

Proof. Let T satisfies property (GR) them $E(T) = \pi a(T) \subseteq \text{iso}\, \sigma(T)$ and by Theorem 2.5 $E(T) = \pi(T)$. Let $\lambda \in \pi^a(T) \Rightarrow \lambda \in \text{iso}\, \sigma_a(T)$ then using [1, Theorem 2.47], $T - \lambda I \in \phi+(X)$ and hence by [13, Theorem 6], $\dim H_0(T-\lambda I) < \infty$.

Conversely, Let $\lambda \in \pi^a(T) \subset \text{iso}\, \sigma(T) \subset \text{iso}\, \sigma_a(T)$, $0 < \alpha(T-\lambda I)$, then $\pi^a(T) \subset E(T)$. Let $\lambda \in E(T)$ T has SVEP at $\lambda$ and using (ii) and [1, Theorem 2.47] $\lambda$ is a left pole of T. Thus $\lambda \in \pi^a(T)$. Thus, $E(T) \subset \pi^a(T)$.

The following example shows that Weyl's theorem and property (GR) are independent.

**Example 2.7.** Let L and R be left shift and right shift operators on $\ell^2(N)$ respectively. Define $T = L \oplus R$, then $\alpha(T) = \beta(T) = 1$ and $p(T) = \infty$. Therefore, $0 \notin \sigma_w(T)$ but $0 \in \sigma_b(T)$. So Weyl's theorem and Browder's theorem do not hold for T . On the other hand $\sigma(T) = D(0, 1)$ so $E(T) = \phi$ and $\sigma_a(T) = \sigma_{ub}(T) = C(0, 1)$, the unit circle. Thus $\pi^a(T) = \phi = \pi_0^a(T)$ Hence, $E(T) = \pi^a(T)$ and $E0(T) = \pi_0^a(T)$. The property (R) and (GR) hold for T .

The following result shows that generalized a-Browder's theorem with property (GR) entails generalized a-Weyl's theorem.

**Theorem 2.8.** If $T \in B(X)$ satisfies both generalized a-Browder's theorem and property (GR), then T satisfies generalized a-Weyl's theorem.

Proof. Since T satis_es generalized a-Browder's theorem and property (GR), therefore,

$\sigma_a(T) \sim \sigma_{SBF_+^-}(T) = \pi^a(T) = E(T) = \pi(T)$

Let $\lambda \notin \sigma_{SBF_+^-}$ and $\lambda \in E^a(T)$ then $\lambda \in \text{iso}\, \sigma_a(T)$ and $T - \lambda I \in \phi_+(X)$, and T* has SVEP at $\lambda$. Thus $\sigma_a(T) = \sigma(T)$. Thus $E^a(T) = E(T)$. Thus, $E^a(T) = \pi^a(T)$. By [7] generalized a-Weyl's Theorem holds for T .

An operator T is said to have property (gb) if

$\sigma_a(T) \sim \sigma_{SBF_+^-}(T) = \pi(T)$

introduced and studied in [11]. The following examples are given in support of Theorem 2.8. The following examples show that property (gb) and (GR) are independent.

**Example 2.9.** Let $T \in B(\ell^2(N))$ be the weighted right shift defined by $T(x_1, x_2, \ldots) = (0, x_{1/2}, x_{2/3} \ldots)$ for all $\{x_n\} \in \ell^2(N)$, $\sigma_a(T) = \sigma_{SBF_+^-}(T) = \{0\}$, $\pi(T) = \phi$. Therefore, $\sigma_a(T) \sim \sigma_{SBF_+^-}(T) = \phi$, $\pi^a(T) = \{0\} = E(T)$. Then T satisfies property (gb) and property (GR) but not generalized a-Browders theorem.

**Example 2.10.** Let $R \in B(\ell^2(N))$ be the unilateral shift and

$U(x_1, x_2, \ldots) = (0, x_2, x_3 \ldots)$

for all $(xn) \in \ell^2(N)$. If $T = R \oplus U$, then $\sigma(T) = D(0, 1)$ so iso $\sigma(T) = E(T) = \phi$. T is polaroid. Moreover, $\sigma_a(T) = C(0, 1) \cup \{0\}$ where $C(0, 1)$ is the unit circle. $\sigma_{ub}(T) = C(0, 1)$. Therefore, $\sigma_a(T) \sim \sigma ub(T) = \{0\} = \pi^a(T)$. But $\pi a(T) \neq E(T)$. Thus T satisfies neither property (gb) nor (GR).

The following result shows if T*has SVEP at every $\lambda \notin \sigma SBF_+^-(T)$. Then the property (GR), generalized a Browder Theorem, property (gw) and property (gab) are all equivalent.

**Theorem 2.11.** Let $T \in B(X)$ has SVEP at every $\lambda \notin \sigma_{SBF_+^-}(T)$. Then the following statements are equivalent:

(i) $E(T) = \pi(T)$

(ii) $E^a(T) = \pi^a(T)$

(iii) $E(T) = \pi^a(T)$

Consequently property (GR), property (gw), generalized a-Browder's theorem, generalized a-Weyl's theorem are equivalent for T .

Proof. By [10, Theorem 2.4] we get
$\sigma(T)=\sigma_a(T)$, $\sigma_{BW}(T)=\sigma_{SBF_{\mp}}(T)$
and
$\pi^a(T)=\sigma_a(T)\sim\sigma_{SBF_{\mp}}(T)$
$=\sigma_a(T)\sim\sigma_{BW}(T)$
$=\pi(T)$
(i)⇒(ii) $E(T)=\pi(T)=\pi^a(T)$ and $E(T)=E^a(T)$.
Thus $E^a(T)=\pi^a(T)$.
(ii)⇒(iii) $E(T)=E^a(T)=\pi^a(T)=\pi(T)$
(iii)⇒(i) $E(T)=E^a(T)=\pi^a(T)$.

Dually, we have

**Theorem 2.12.** Suppose that T has SVEP at $\lambda\notin\sigma SBF_{\mp}(T)$. Then the following statements are equivalent.
(i) $E(T^*)=\pi(T^*)$;
(ii) $E^a(T^*)=\pi^a(T^*)$;
(iii) $E(T^*)=\pi^a(T^*)$;

Consequently, property (GR), property (gw), generalized a-Brawder's theorem, generalized a-Weyl's theorem. are equivalent for T*

## 3. Property (GR) for Polaroid Operators

This section is devoted to the classes of operators for which the isolated points of the spectrum are poles of the resolvent. We know that if T* has SVEP, then $\sigma(T)=\sigma_a(T)$

Therefore,
T a polaroid ⇔ T polaroid.
If T has SVEP, then
T* a polaroid ⇔ T* polaroid ⇔ T polaroid.

**Theorem 3.1.** If $T\in B(X)$ is a-polaroid, then T satisfies property (GR).

Proof. Let $\lambda\in\pi^a(T)$ then $\lambda\in\text{iso}\,\sigma_a(T)$. Since T is a-polaroid, therefore λ is a pole of the resolvent and hence an isolated point of $\sigma(T)$. Also $0<\alpha(T-\lambda I)$. Then, $\lambda\in E(T)$. Conversely, let $\lambda\in E(T)$ then $\lambda\in\text{iso}\,\sigma(T)\subset\text{iso}\,\sigma_a(T)$. Thus, λ is a pole of the resolvent of T . Hence $\lambda\in\pi^a(T)$. Hence
$E(T)=\pi^a(T)$.

The Theorem 3.1 does not hold in case of weaker condition of being T polaroid. This is shown in the following example.

**Example 3.2.** Let $R\in B(\ell^2(N))$ be the unilateral right shift defined as
$R(x_1, x_2, \ldots) := (0, x_1, x_2, \ldots)$ for all $(x_n)\in\ell^2(N)$,
and
$U(x_1, x_2, \ldots) := (0, x_2, x_3, \ldots)$ for all $(x_n)\in\ell^2(N)$.
If $T := R\oplus U$ then $\sigma(T)=D(0.1)$, so iso $\sigma(T)=\phi$.
Moreover, $\sigma_a(T)=C(0,1)\cup\{0\}$, $C(0,1)$ is the unit circle, so iso $\sigma_a(T)=\{0\}$.$p(T)=P(R)+P(U)=1$. 0 is a left pole. But $q(T)=q(R)+q(U)=\infty$.

So T is not a-polaroid, T is polaroid.
$E(T)=\phi$, $\pi^a(T)=\{0\}$. Thus property(GR) is not satisfied.
Let $T\in B(X)$ and let $f\in H(\sigma(T))$, where if $H(\sigma(T))$ is the space of functions analytic in an open neighborhood of $\sigma$ (T), Such that f is non-constant on each of the components of its domain. Define by the classical functional calculus, f(T) for every $f\in H(\sigma(T))$

**Theorem 3.3.** Suppose that $T\in B(X)$ is polaroid and $f\in H(\sigma(T))$.
(i) If T* has SVEP, the property (GR) holds for f(T), or equivalently property (gw), generalized a-Weyl's theorem, generalized a-browner's theorem hold for f(T).
(ii) If T has SVEP, then property (GR) holds for f(T*), or equivalently property (gw), generalized a-Weyl's theorem, generalized a-Browder's theorem hold for f(T*).
Proof. (i) By [2, Theorem 3.11], f(T) is polaroid and f(T*) has SVEP [1,Theorem 2.40]. Hence, f(T) is a-polaroid. By Theorem 3.1 f(T) has property(GR). Using Theorem 2.11, property (gw), generalized a-Weyl's theorem, generalized a-Broader's theorem hold for f(T).

(ii) We know T*is polaroid and hence f(T*) is polaroid [2, Lemma 3.11],
f(T) has SVEP [1, Theorem 2.40] and f(T*) is a-polaroid, By Theorem 3.1,
f(T*) has property (GR). Thus by Theorem 2.12, property (gw), generalized
a-Weyl's theorem, generalized a-Browder's theorem hold for f(T*).

## References


[1] P. Aiena, Fredholm and Local Spectral Theory with Applications to Multipliers, Kluwer Acad. Pub., 2004.

[2] P. Aiena, E. Aponte and E. Bazan, Weyl Type theorems for left and right polaroid operator, Int. Equa. Oper. Theory, 66(2010), 1-20.

[3] P. Aiena, J. Guillen and P. Pena, Property (w) for perturbations of polaroid operators, Linear Algebra and its Appl., 428(2008), 1791-1802.

[4] P. Aiena, J. R. Guillen and P. Pena, Property (R) for bounded linear operators, Mediter J. Math. (2011), preprint.

[5] P. Aiena and P. Pena, A variation on Weyl's theorem, J. Math. Anal. Appl., 324(2006), 566-579.

[6] P. Aiena and F. Villfane, Weyl's theorem for some classes of operators, Int. Equa. Oper. Theory, 53(2005), 453-466.

[7] P. Aiena and T. L. Miller, On generalised a-Browder's theorem, Studia Math., 180 (3)(2007), 285-300.

[8] M. Amouch, Polaroid operators with SVEP and perturbations of property (gw), Mediter. J. Math., 6(2009), 461-470.

[9] M. Amouch and M. Berkani, On the property (gw), Meditor. J. Math., 5(2008), 371-378.

[10]M. Berkani, M. Sarih and H. Zariouh, Browder type theorems and SVEP, Mediter J. Math., (2010), 399-409.

[11]M. Berkani and H. Zariouh, New extended Weyl type theorems, Math. Vasnik, 62(2010), 145-154.

[12]K.B. Laursen and M.M. Neumann, Introduction to local spectral theory, Clarendon Press, Oxford, 2000.

[13]V. Rekocevic, On a class of operators, Math. Vesnik, 37(1985), 423-426.

[14]C. Schmoeger, On isolated Points of the spectrum of bounded linear operators, Proe Amer. Math. Soc., 117(3) (1993).